THE DECAY $K_L \to \pi^\circ\gamma\gamma$

M.K.Volkov, M.Nagy*

By using the linear σ-model the estimates of the branching ratio of the decay $K_L \to \pi^\circ\gamma\gamma$ have been obtained. The comparison with the experimental data of the E731 group has been made.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Распад $K_L \to \pi^\circ\gamma\gamma$

М.К.Волков, М.Надь

В рамках линейной σ-модели были получены оценки вероятности распада $K_L \to \pi^\circ\gamma\gamma$. Было проведено сравнение с данными экспериментальной группы E731.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

The authors of recently published paper /1/ have performed a search for the rare decay mode $K_L \to \pi^\circ\gamma\gamma$ using a data set from E731 experiment. This decay is of interest for its contribution to the decay $K_L \to \pi^\circ e^+ e^-$, which gives some possibilities of observing a direct CP violation. They have concluded, using world averages for $K_L \to 2\pi^\circ$ and for $K_L \to 3\pi^\circ$ branching ratios /2/ that $B(K_L \to \pi^\circ\gamma\gamma) < 2.7 \times 10^{-6}$ (90% confidence level).

In 1982 one of the authors of the present paper calculated the width of the decay $\eta \to \pi^\circ\gamma\gamma$ /3/ in the framework of the quark model of superconductivity type (QMST) /4,5/, just proposed at that time. This model is a quark analogy of the well-known model of Nambu — Jona-Lasinio /6/. Starting from the effective chirally symmetric four-quark interaction, one can obtain in QMST the well-known chiral phenomenological Lagrangians, describing the linear σ-model and the interactions of scalar, pseudoscalar, vector and axial vector mesons. The number of free parameters in this case is less than that in the old phenomenological approaches.

---

*Institute of Physics, EPRC, Slovak Academy of Sciences, CS-842 28 Bratislava, Czechoslovakia.

The advantages of the linear σ-model compared with the nonlinear variant one can realize at the description of the process $\eta \to \pi^{\circ}\gamma\gamma$. The experimental status of this process was unclear for quite a long time. Before the beginning of eighties the experimental value of the decay rate of this process was considered as equal to the decay rate of the process $\eta \to \pi^{+}\pi^{-}\gamma$. We have shown already in 1979 the uncorrectness of such experimental results from the point of view of phenomenological chiral Lagrangians /7/. Really, the experiments, performed in IHEP (Serpukhov) in 1980-1981, have confirmed, that the width of the decay $\eta \to \pi^{\circ}\gamma\gamma$ is almost 70-times smaller, than that of the decay $\eta \to \pi^{+}\pi^{-}\gamma$ /8/.

From the theoretical results, obtained before the IHEP experiments, the closest to the experimental data were the calculations published by Oppo and Oneda /9/. They have accounted the contributions of the vector mesons in the process $\eta \to \pi^{\circ}\gamma\gamma$ $(\eta \to \gamma[(\omega, \rho, \phi) \to \pi^{\circ}\gamma])$. However, their result was triple smaller than the next experimental result. In the nonlinear chiral theory the large contributions are also given by the diagrams depicted in Fig.1

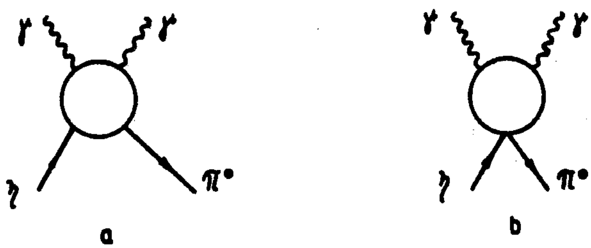

Fig.1

but, these contributions are mutually compensated. Quite a different situation takes place in the linear σ-model with the real intermediate $a_0(980)$ meson (in the old notations $\delta(980)$) in the diagram of the type 1b. In QMST these loop diagrams have the form shown in Fig.2.

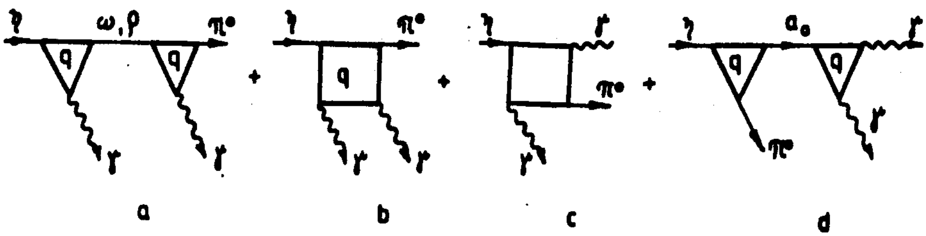

Fig.2

Here the diagram 2d also partially compensates the lagre contributions of the diagrams 2b and 2c. The remaining part, in contrast to the non-linear model, is nonvanishing and completely commensurate with the contribution of the diagram 2a, having ω and ρ mesons in the intermediate state. As a result we have obtained for the width $\Gamma_{\eta\to\pi^\circ\gamma\gamma}$ the value equal to 1 eV, that is in good agreement with the experimental data /8/.

The contributions of the contact and pole diagrams with an intermediate scalar meson have been accounted in the amplitude of the decay $\eta\to\pi^\circ\gamma\gamma$ for the first time in paper /10/. There the author got the result quite close to the future experimental data /8/. Then, already after the new experimental data appeared, the similar numerical estimates have been obtained in /11/. However, by comparing the results of the paper /10/ with those of /3,9,11/ one can realize that in /10/ the estimates of the contributions of the pole diagrams with vector mesons into the width of the decay $\eta\to\pi^\circ\gamma\gamma$ have been twice higher. Therefore the agreement of the final result with the recent experimental data has only the accidental character. In the articles /3/ and /11/, the results of which are completely mutually consistent, the contributions of these diagrams are accounted more correctly.

After these general remarks we overcome to the calculation of the amplitude of the decay $K_L\to\pi^\circ\gamma\gamma$ that can be easily expressed through the amplitude of the decay $\eta\to\pi^\circ\gamma\gamma$. Really, the process $K_L\to\pi^\circ\gamma\gamma$ is completely defined by the three pole diagrams, depicted in Fig.3.

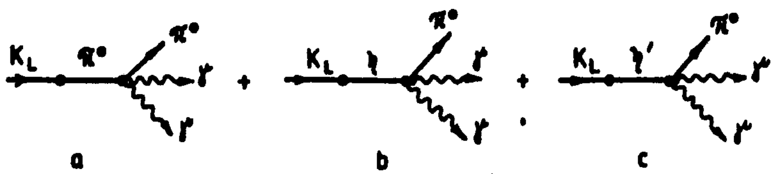

Fig.3

However, the dominant contribution to the amplitude $K_L\to\pi^\circ\gamma\gamma$ will give the diagram 3b, because there is the pole very close to the resonance region. The diagrams 3a and 3c are not only far from the resonance, but even are partially compensated, having opposite signs. That is why we shall neglect their contributions in the following.

The amplitude of the decay $K_L\to\eta\to\pi^\circ\gamma\gamma$, taking into account the previous arguments, can be expressed in the form

$$T_{K_L\to\pi^\circ\gamma\gamma} = \frac{c}{m_\eta^2 - m_{K_L}^2} T_{\eta\to\pi^\circ\gamma\gamma} = a\,T_{\eta\to\pi^\circ\gamma\gamma} , \qquad (1)$$

where m_η and m_{K_L} are masses of the η and K_L mesons, respectively.

The quantity a, connected with the weak vertex, defining the probability of the transition $K_L \to \eta$, is considered as a free parameter. We define this parameter from the independent experimental data, namely from the widths of the decays $K_L \to \gamma\gamma$ and $\eta \to \gamma\gamma$. The amplitudes of these decays can be expressed analogously to (1)

$$T_{K_L \to \gamma\gamma} = \frac{c}{m_\eta^2 - m_{K_L}^2} T_{\eta \to \gamma\gamma} = a\, T_{\eta \to \gamma\gamma} . \tag{2}$$

For the amplitude $T_{\eta \to \gamma\gamma}$ we get from the recent experimental data /2/ the value

$$\begin{aligned} \Gamma^{exp}_{\eta \to \gamma\gamma} &= \frac{m_\eta}{\pi} \left(T^{exp}_{\eta \to \gamma\gamma} \frac{m_\eta}{8}\right)^2 = 0.42 \text{ keV} , \\ T^{exp}_{\eta \to \gamma\gamma} &= \frac{8}{m_\eta} \left(\frac{\pi\, 0.42 \text{ keV}}{m_\eta}\right)^{1/2} = \frac{0.0124}{m_\eta} . \end{aligned} \tag{3}$$

Then, by using (2), we have the following formula for the width of the decay $K_L \to \gamma\gamma$

$$\Gamma_{K_L \to \gamma\gamma} = \frac{m_{K_L}}{\pi} \left(a\, T_{\eta \to \gamma\gamma} \frac{m_{K_L}}{8}\right)^2 = 7.24 \times 10^{-12} \text{ eV} . \tag{4}$$

From here we can easily evaluate the parameter a

$$a = 1.5 \times 10^{-7} . \tag{5}$$

Now we have all necessary aids for the calculation of the width of the decay $K_L \to \pi^\circ\gamma\gamma$

$$\Gamma_{K_L \to \pi^\circ\gamma\gamma} = a^2\, \Gamma_{\eta \to \pi^\circ\gamma\gamma} (m_\eta \to m_{K_L}) . \tag{6}$$

The evaluation of the contributions of diagrams in Fig.2 into the width of the decay $K_L \to \pi^\circ\gamma\gamma$, with the change of the mass of η-meson to the mass of K_L-meson, leads to the following results.

The contribution of the diagram with the intermediate vector meson (Fig. 2a) equals

$$\Gamma^{(\rho,\omega)}_{K_L\to\pi^o\gamma\gamma} = \frac{\pi}{2} M \left(\frac{M}{8\pi^2 F_\pi}\right)^4 [3\alpha\alpha_\rho a \sin\bar{\theta}]^2 [I_{\rho\rho} + 2I_{\rho\omega} + I_{\omega\omega} = \tag{7}$$

$$= 4.8\times 10^{-3}] = 0.5\times 10^{-14} \mathrm{eV},$$

where M denotes the mass of the K_L-meson, $F_\pi = 93$ MeV is the pion decay constant, $\alpha = e^2/4\pi = 1/137$ is the electromagnetic constant, $\alpha_\rho = g_\rho^2/4\pi = 3$ is the constant of the decay $\rho\to 2\pi$, $\bar{\theta} = \theta_0 - \theta$, $\theta_0 \approx 35°$ is the ideal mixing angle, $\theta = -18°$, $I_{\rho\rho}$, $I_{\omega\omega}$ and $I_{\rho\omega}$ are the phase integrals, calculated in the paper /9/. We have used the values of these integrals obtained after replacing the mass m_η by the mass m_{K_L}.

The contribution of the diagrams 2b — 2d is given as follows /3/

$$\Gamma^{(\Box+\delta)}_{K_L\to\pi^o\gamma\gamma} = \frac{M}{9\pi}\left(\frac{m}{2\pi F_\pi}\right)^4 (\alpha a \sin\bar{\theta})^2 \times \tag{8}$$

$$\times \int_1^{\frac{M^2+m^2}{2mM}} dx (x^2-1)^{1/2} \left(\frac{M^2+m^2}{2mM} - x\right)^2 \left(1 - \frac{4m_u^2}{m_{a_0}^2 - M^2 - m^2 + 2mMx}\right)^2,$$

where m is the mass of the π^o-meson, m_{a_0} is the mass of the a_0-meson and m_u is the mass of the constituent u and d quarks, having in our model the value /5/

$$m_u^2 = \frac{m_{a_1}^2}{12}\left[1 - \left(1 - \left(\frac{2g_\rho F_\pi}{m_{a_1}}\right)^2\right)^{1/2}\right]. \tag{9}$$

Here m_{a_1} is the mass of the axial-vector a_1-meson. For the value $m_{a_1} = 1270$ MeV /10/ we get $m_u \approx m_d \approx 280$ MeV and from the formula (8) we get the value

$$\Gamma^{(\Box+\delta)}_{K_L\to\pi^o\gamma\gamma} = 0.34\times 10^{-14}\ \mathrm{eV}. \tag{10}$$

The last important term, giving the sizeable contribution into the decay $K_L\to\pi^o\gamma\gamma$, is coming from the interference of the diagrams of the type 2a and 2b — 2d. This term has the following form

$$\Gamma^{\text{interf}}_{K_L\to\pi^\circ\gamma\gamma} = \frac{a_\rho M}{2}\left(\frac{\alpha a \sin\bar\theta}{\pi}\right)^2\left(\frac{m}{2\pi F_\pi}\right)^4\left(\frac{m_\rho^2}{2mM} I_1 - I_2\right) = \tag{11}$$

$$= 0.54\times 10^{-14}\ \text{eV},$$

where

$$I_1 = \int_1^{\frac{M^2+m^2}{2mM}} dx\left(\frac{M^2+m^2}{2mM} - x\right)^2\left(1 - \frac{4m_u}{m_{a_o}^2 - M^2 - m^2 + 2mMx}\right)\times \tag{12}$$

$$\times \ln\frac{m_\rho^2 - Mm(x - \sqrt{x^2-1})}{m_\rho^2 - Mm(x + \sqrt{x^2-1})},$$

$$I_2 = \int_1^{\frac{M^2+m^2}{2mM}} dx\sqrt{x^2-1}\left(\frac{M^2+m^2}{2mM} - x\right)^2\left(1 - \frac{4m_u}{m_{a_o}^2 - M^2 - m^2 + 2mMx}\right). \tag{13}$$

The sum of all these contributions leads to the following value of the width of the decay $K_L\to\pi^\circ\gamma\gamma$

$$\Gamma_{K_L\to\pi^\circ\gamma\gamma} = 1.4\times 10^{-14}\ \text{eV}, \tag{14}$$

or to the value of the branching ratio

$$B(K_L\to\pi^\circ\gamma\gamma) = \frac{\Gamma_{K_L\to\pi^\circ\gamma\gamma}}{\Gamma_{K_L\to \text{all}}} = 1.1\times 10^{-6} \tag{15}$$

which does not contradict the experimental data on the upper bound of the branching ratio published in /1/.

The estimates of the width of the decay $K_L\to\pi^\circ\gamma\gamma$ becomes an especially actual meaning at present time, when LEP at CERN is starting to operate. In particular, there are in preparation some new experiments with neutral particles as the final decay products. The test of our theoretical results could undoubtedly provide the interesting information concerning the confirmation of the advantages of using the linear version of σ-model in the elementary particle physics. The result of /1/ is close to our estimates and there is the possibility of measuring the upper bound in the framework of LEP activities.

References

1. Papadimitriou V. et al. (E731). — Phys. Rev. Lett., 1989, 63, p.28.
2. Particle Data Group, Phys. Lett., 1988, B204, p. 1.
3. Volkov M.K., Kreopalov D.V. — Yad. Fiz., 1983, 37, p. 1297.
4. Volkov M.K., Ebert D. — Yad. Fiz., 1982, 36, p. 1265; Z.Phys., 1983, C16, p. 205.
5. Volkov M.K. — Ann. Phys., 1984, 157, p. 282; Part.Nuclei, 1986, 17, p. 433.
6. Nambu Y., Jona-Lasinio G. — Phys. Rev., 1961, 122, p. 345.
7. Volkov M.K. — Part. Nuclei, 1979, 10, p. 693.
8. Binon F. et al. — Yad. Fiz., 1981, 33, p. 1534.
9. Oppo G., Oneda S. — Phys. Rev., 1967, 160, p. 1397.
10. Gounaris G.J. — Phys. Rev., 1970, D2, p. 2734.
11. Ivanov A.N., Troitskaya N.I. — Yad. Fiz., 1982, 36, p.494.

Received on December 15, 1989.